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Volume 7: Science and Philosophy

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Book 2: Scientific Method

Chapter 4: Notes on Science

§3. Measurement

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§3. Measurement †13

280. *Definition.* A *character* is a possible fact regarded as concerning a particular thing or things.

Illustration. The moon may come between the sun and the earth so as to cast a shadow upon the latter. That is a possible fact. Now, if we think of this fact as something that concerns, or modifies what we can say of, the sun, we call it an eclipse of the sun; and so considered, it is what we mean by a *character* of the sun at such an instant.

281. *Explanation.* I say a *possible* fact, because if a character is not actually true of a given thing, that is not sufficient to prevent its being a character. Thus, when the sun is not eclipsed, it does not possess that character, but we do not say that there is no such character. If it could be shown that the supposition of the moon coming between the sun and the earth so as to cast a shadow on the latter, could only be carried out in ways which, being examined, would prove all of them to involve contradictions, then we should say this is no real character, but only a phrase, which cannot be realized in any imaginable manner. In short, the character in itself does not pretend to belong to the world of experience; to be a character it only needs to have a place in the realm of ideas. When the character is attributed to any particular thing, that thing is something having its place in experience. It is not needful, for the purposes of mathematics, to inquire particularly what experience is. It may be said, however, that it is something which is forced upon us; so that one element of it is its insistence, whether we like it or not. And it is also something which forces itself not merely momentarily upon me; but upon me and you alike, at various times, so that it has a certain consistency and extension in its forcefulness. Finally, it is something of which our knowledge can never be complete; so that there is always a difference between the experienced thing and our idea of it. But since the character, to be a character, need not really belong to the particular thing, but it is sufficient if we can really ask whether or not it belongs to that thing, it will be seen that we do not need to go very far into a study of its nature.

282. *Analysis.* Characters may be more or less precisely defined. A general character may be conceived as a multitude of precise characters. Mathematical thought consists in the study of precise relationships between ideal objects. But a possible fact may vary in an indefinite multitude of different ways. The moon coming between the earth and the sun, may not only have a multitude of different positions, but it may have a multitude of different shapes and colors and chemical compositions; and so may the sun and earth. For the purposes of mathematics, it is necessary, in the first place, to abstract from most of those differences, and consider them as insignificant and, for our purposes, nil. Then, those varieties which still, for our purposes, are different, have to be arranged, as far as possible, dimensionally. To do this, we imagine any one precise variety of the character, — precise, I mean, after the proper abstractions have been made, — and conceive that as undergoing a multitude of variations in time, from the infinitely distant past to the infinitely distant future, such that in the course of all time it will be continually changing; and we prefer to take this series so that the character will ultimately return to its initial state, and just barely return to that state. If this series of changes does not include all the variations, we think of the whole multitude of states through which the character passes in all time, as belonging at one instant to an equal multitude of objects; and then we conceive the characters of those objects to undergo in the whole course of time a continuous series of changes of a different kind, so as to be entirely distinct. We call this a second dimension, or series of variations. We can make any number of these dimensions; and as far as possible we thus seek to give some precise arrangement to all the variations of characters. When this method fails us, we can resort to other systems of arrangement, of some of which we shall have examples in geometry.

283. Any one of those dimensions is such that the characters can pass through the whole series of states in the course of time. It is easy to imagine multitudes of variations so related to one another that one precise character could not even in all time pass through them all by insensible gradations. For example, colors differ from one another, not merely in hue, but also in luminosity, and in chroma, or intensity of departure from grey. Now, starting with a color of some precise hue, precise luminosity, and precise chroma, if it is to change its hue gradually, for each precise hue that it takes, it will have just one sole luminosity and one sole chroma; so that, when it has gone through the whole cycle of hues, it will have had for each of them but one single luminosity and but one single chroma. Though it should pass through the cycle of hues times without end, it would still not have begun to exhaust the possible luminosities and chromas for each hue. The student may admit that it might be possible (and, in fact, it might be shown to be possible) that if the color were so to jump from hue to hue, from luminosity to luminosity, from chroma to chroma, that taking any two instants, no matter how near to one another, if during the interval between them the variations should embrace the whole cycle of hues, the whole range of luminosity, and the whole range of chroma, then the color might in the course of time precisely assume for an instant every special variety of color. But if the variation is to take place *gradually,* then it is not possible that the color should in the course of time assume every possible variation.

284. The meaning of this seems to be clear. That is, it possesses the first grade of clearness of ideas, that of containing no element which perfect familiarity does not enable us to use with entire confidence.[†14] But that grade of clearness is not sufficient for precision of statement, and logical security. For that purpose, we must say what we mean by "gradually." In attempting to state this, it first occurs to us to say that we mean by a gradual change of hue, such a change that in passing from one exact hue to another we pass through all intermediate hues. There are two reflections to be made upon this statement. First, it supposes that the different hues are so related in our minds that we are able to say what ones are, and what ones are not, intermediate between any given pair of hues. That is to say, we must have a precise idea of what it means to say that the hues are mentally arranged in a line. But if that be so, we need not introduce the conception of a change in time; for that was only a device to enable us to describe what we mean by a line of variations of character. In truth, though the introduction of the idea of time gives sensuous clearness to our idea, it contributes not in the least to logical clearness. The second reflection which has to be made upon our attempt to define gradual change of hue is that the hues form a circle, the so-called color-circle; so that it is possible to pass from any one to any other by going either way round the circle; and thus there is no particular hue that we need pass through. To define a linear arrangement, the line being permitted to return into itself, it is necessary to speak of four points on the line.

285. *Definition.* A *state* is an exact character, that is, one which, certain understood abstractions being made, admits of no varieties.

286. *Definition.* A *line of variations of states* is a continuous multitude of states such that, taking any four of them all different, there are in the nature of the characters two that are adjacent to any one, or else it would be so were two that are extremes made adjacent to one another; and any fifth character of the same line occupies in its nature a definite position between two adjacent characters of the first four.

287. *Definition.* A *circuit of states* is a line of variation of states which returns into itself and has no extreme states.

Illustrations. Suppose a light to be increasing in intensity from that of a fire-fly toward that of the planet Venus at such a rate that it would at a certain instant, say Midnight, attain that brightness. Suppose next that it increases at a more and more rapid rate, so that after a while its rate of brightening is such that at that rate it would attain the brightness of Venus at 11 o'clock. Suppose it continues to increase more rapidly, so that after a while, its rate of brightening is such that it would attain the brightness of Venus at 10 o'clock. Finally, suppose that its increase becomes so rapid that at that rate it would become as bright as Venus at 9 o'clock. Thus, there is this order among these characters: such a change of brightness as would make it equal Venus at 12, at 11, at 10, at 9. But when it was increasing at a rate to make it equal Venus at 12, its increase might become slower and slower so that the time at which it would equal Venus would become later and later. If it ceased to increase, the time would be thrown into the indefinite future. If it began slowly to decrease, its change would be as if it had been as bright as Venus a long time before. If it decreased faster, this time might become nearer and nearer, until at some time after 9, it was decreasing as if it had been as bright as Venus at 9 o'clock. At a little after 10, it might be at ten o'clock, at a little after 11, at 11 o'clock, and at a little after 12, at 12 o'clock. Thus the order would be reversed. But there is no imaginable way in which the time could gradually change from 12 o'clock to 10 o'clock without passing through either 11 o'clock or 9 o'clock. This shows that the instants of time form a circuit.

288. *Definition.* A *Quality* is that character of a character which consists in its belonging to a particular line of variation of states.

Illustrations. Thus, temperature, probability, wealth, the happening earlier or later than something else are qualities.

289. *Scholium.* The order of states in a line of variation may be shown by attaching to sensibly different states different numbers. For if the line of variation forms a circuit, its states are related to one another like the real numbers, rational and irrational, positive and negative, including ∞, except that the states may perhaps be so multitudinous that it is impossible to assign distinct numbers to them all. Whether any example can be given of a quality in which there are sensible variations too multitudinous for numbers to discriminate or not, there can be no doubt that such a quality might exist.[†15]

290. In mathematics, we have to deal with an ideal condition of things. We imagine ourselves to be in possession of a general method of working by which definite states can be assigned, in the first place, to all rational numbers, in their order. That is, the states in their own nature shall have the same order of succession as the values of the rational numbers. We then suppose states for the irrational numbers to be interposed in their orders. States which the particular method of assignment of numbers may leave unnumbered, between the numbered states, are for this method not sensibly different from the irrational numbers that are near them. A new distribution of numbers by a different method might possibly distinguish some of these, and in doing so it might, or might not, leave others undistinguished.

291. The numbers may occur in every assignable part of the circuit, or may be contained between two limits, or a part of the series of numbers may cover the whole circuit. In the last case, we suppose the remaining numbers to be assigned to the circuit taken over and over again in regular arithmetical progression. In the second case, we are at liberty to fill up the vacant part of the circuit with a second series of numbers which will be distinguished by having a quantity not a number added to it. But in doing this, we shall assume that the numbers are so assigned that taking any three states A, B, C, a state, D, can be found whose number diminished by that of C equals the number of B diminished by that of A.

292. *Definition.* A *method of* ***measurement*** upon a circuit of variation is a general rule according to which, it is possible to assign each rational number to an exact state and to but one; and conversely, given the numbers assigned to any three states, it is possible to ascertain whether any fourth rational number is exactly assignable to a given state or not, and if not in what one of the four intervals between the four numbers, the number that ought to be assigned to the given state falls. [*Corollary.* This affords the means of vaguely assigning states to the irrational numbers.][†16] Numbers so assigned to states may be called *state-numbers.* States to which definitely different places in the scale of numbers cannot be assigned are said not to be *measurably* distinct, according to the particular system of **measurement** employed. If the whole circuit of numbers does not precisely correspond to the whole circuit of states, it is assumed that either, on the one hand, the same state receives different numbers or, on the other hand, that different states receive the same numbers distinguished by the addition of a non-numerical quantity. But in all such cases the numbers are to be so distributed that if any three states have a, b, c for state-numbers not infinite (in the same system of **measurement**) then there shall be one state and but one which has for a state-number, $a + b - c$.

293. *Scholium.* Were we to say, at once, that there is a state for every syzygy of two state-numbers, we should no longer have a line of variation, in case a non-numerical increment is required for part of the circuit.

294. *Scholium.* Suppose we have two objects both capable of taking states of the same quality and of changing those states, but only in the inherent order of the circuit of variation. Suppose further that the whole pair has but a single degree of freedom of changes, so that for each state of the one there is one and but one state of the other. Finally, suppose that one member of the pair can always be changed to the state which before the change was that of the other. If, then, we affix numbers to the states in such a way that every change in the state-number of one of the pair shall be equal to that of the other, these numbers will fulfil the

necessary conditions. But if there is a region over which the pair cannot be moved, then it will be necessary to have as many such changeable pairs as there are regions. Each pair may conveniently consist of one object in one region and one in another.

295. Another method of **measurement** would be obtained if we had a multitude of objects so that there was just one for each state of the quality in the whole circuit, and if the whole multitude had but a single degree of freedom of change and two positions of the whole multitude were given.

296. Of course, it would be still more convenient if we had given all states of such a multitude of objects. But in that case, they must conform [to] the condition that, let A, B, C, D, be what different states they may, if a change of one member of the multitude from A to B is synchronous with the change of another member from C to D, then every change of any member from A to B is simultaneous with the change of the member in state C to D.

297. *Definition.* A *standard of* ***measurement*** upon a circuit of variation is a pair of objects subject to the quality of the circuit, such that either can be changed to the state of the other in respect to that quality, but all its changes must follow the inherent order of the states, and the whole pair has but one degree of freedom of position, so that if one of the pair is in any given state there is but one state in which the other can be; and the method of **measurement** is such that station-numbers of the states of the two objects change by equal differences simultaneously.

298. *Definition.* A *metron* is an object composed of parts in one to one correspondence with all the states of a circuit of variation, such that any two form a standard of **measurement** , and the whole has *steady displacements,* that is, if the change of one part from state A to state B is synchronous with the change of the part at first in state C to state D, then the change of any other part from state A to state B is synchronous with the change of the part at first in state C to state D.

299. *Definition.* The *quantity,* or *modular quantity,* of an interval between two states in a line of variation is that which there is in the inherent difference between two states which justifies their receiving state-numbers whose difference has this or that amount. Otherwise, it is that whose *measure* is the difference of their state-numbers multiplied by a unit expressive of the unit of difference.

300. *Division.* **Measurement** is either

A. *Parabolic,* when the circuit of real numbers corresponds to the circuit of variation of states, or

B. *Non-parabolic,* when the two circuits are not coincident. *Non-parabolic* **measurement** is of two kinds, viz.: — a. *Elliptic,* when the entire circuit of states coincides with a finite part of the circuit of numbers. b. *Hyperbolic,* when the entire line of finite numbers occupies but a portion of the circuit of variation, and leaves a portion vacant.

301. *Definition.* An *absolute,* or *firmamental,* state is a state to which or from which no member of the metron can change.

302. *Theorem. In parabolic* ***measurement*** *upon one circuit there is just one firmamental state.*

Demonstration. In every measurable change of the metron, the station-numbers of all members are changed by the same finite increment. Now, a finite number added to any number gives a different number with one sole exception. Namely, if the number increased is infinite, the sum has the same value. But only one state has a given state-number. Therefore, the state of that member of the metron which has ∞ for its state-number is never changed. Hence, since every change can be reversed, no member is by any measurable change carried to the state whose state-number is infinite.

303. *Theorem. In elliptical motion there is no firmamental state, and the modular quantity of the whole circuit is finite, and the* ***measurement*** *proceeds in one annular order.*

Demonstration. If the entire circuit of variation is covered by a finite part of the scale of numbers, let the metron receive any measurable change. Then, the state-numbers of all its members will receive the same finite increment. Consequently, those members whose state-numbers were sufficiently great will be made greater than the finite difference of numbers of the whole circuit, and will be made greater than any of the numbers were at first. Since no state-number is infinite no member will have its state unchanged. And every member will have an arithmetical progression of state-numbers, the difference of all these progressions being the same. That difference will be the modular quantity of the circuit.

304. *Theorem. In hyperbolic motion there are just two firmamental states, and in both the regions into which they sever the circuit the state-numbers increase toward one of these and away from the other; and the quantity of the whole circuit is not infinite.* . . .[†17]

305. *Analysis.* Our measures of space are of various descriptions. Some of them, as square measure, cubic measure, etc. are composite. Thus, square measure depends upon long measures of length and breadth. Other kinds of measure are derivative. Thus, the distance of a point from a surface is the distance of the point from the nearest point of the surface. The measures which are neither composite nor derivative are two: long measure between pairs of points and angular measure between pairs of planes.

306. There are many different sorts of standards which might be assumed for long measure. From the point of view of theoretical physics the average distance that a molecule of hydrogen at a standard temperature would move in a day would have something to recommend it. But the primitive and most usual sort of standard, as well as the most convenient for the purposes of geometry, is a rigid body which is carried about from place to place, such as a yard-stick.

307. Let us ask what we mean by a rigid body. If we say that it is a body whose measures remain fixed, somebody may reply that if it is taken as the standard of measure it means nothing to say that its measure relatively to itself remains the measure of itself. Some care is, therefore, required in saying what we mean by a rigid body. If a rigid body were a single body there would be less meaning in the word rigid than there is when it is a kind of body which the working of Nature makes to be a usual kind, or a kind toward the ideal properties of which many objects closely approximate. Our general experience leads us to think that solid bodies come very near to being bodies which at fixed temperature and free from external influences have certain properties which we proceed to consider. First, every particle of such a body, or part of it occupying an indivisible place, always occupies a single indivisible place. It never goes out of existence or out of space; nor does it separate so as to occupy a number of points, nor does it enlarge so as to occupy a line or a surface or a solid space. Second, every flat film of such a body, or part of it occupying a plane, always continues (there being no stress upon the body) to occupy a plane. It neither breaks nor bends. Third, in any straight fibre of such a body, or part of it occupying a ray, (and which must always continue to occupy a ray, since every flat film in which it lies continues to lie in a plane) the order and continuity of the particles remain always the same, and round such straight fibre the order and continuity of the flat films through it remain always the same. These three properties, taken together, may be termed the optical property of the rigid body.

308. In the next place, our general experience leads us to believe that a perfectly rigid body, which ordinary solid bodies sensibly resemble, is such that if a given particle of it be brought to a point [M] and the straight filament through that particle and a

second particle be brought into a given ray through [M], then there are just two points to which that second particle can be brought. In like manner, if of three particles two are brought to two points [M] and [N], there are, in each plane through {MN} just two points to which the third particle might be brought. But there is a difference between the two cases. Namely, as long as the straight filament remains in the ray, if at one time one particle is at the point [M] and another at the point [N], then, no matter how the filament is moved in the ray, every time the first particle is brought back to [M] the second particle can be brought to no other point than [N]. But in the plane, though the flat film remain in all its movements in that plane, yet if the first particle is at first at [M] the second at [N] and the third at [P], if the first two particles are carried along {MN} until they have performed an entire circuit, while the third always remains in the plane, it will not return to the same point [P] (supposing it has ever left that point) but will return to a fourth point [Q] and it will not be until a second circuit has been performed that it returns to [P]. We shall demonstrate that this must be so. It is mentioned now for a special purpose. We shall find the difference is owing to the ray having an odd number, and the plane an even number of dimensions. Hence, in three dimensions if of four particles of a rigid body three are restored to their initial positions, there is only one point to which the fourth can be brought, although there is another point which it could perfectly well occupy (as shown by its looking glass image) if there were only any fourth dimension through which it could be carried to that point. On this account, it is convenient in geometry to imagine ourselves to be in possession of a rigid body to be used as a standard of **measurement** which shall have the purely imaginary property that, while any three of its particles remain fixed, it shall be capable of being *perverted,* that is, of being suddenly converted into its looking glass image. On account of this property, we will denominate the thing, not a rigid body, but a rigid image.

309. Finally, we are led to believe that if any number of parts of a rigid body occupy at one time certain positions and at another time certain other positions, then any other parts which may at any time occupy the first set of positions may be carried into the second set of positions.

310. There is another property of rigid bodies which makes all long measure to be parabolical and all angular measure to be elliptical; but it is best, at first, to consider the consequences of the other properties, and to reserve the consideration of this until later.

311. *Definition.* A *metrical image* is a continuous multitude of particles, straight filaments, and flat films having the following properties:

1st, it has the optical property that whatever part occupies at one instant a point or a plane occupies the same kind of place (whichever it may be) at every instant, and with the same connective relations to straight filaments.

2nd, the fixation of a particle of such image at a point or of a flat film in a plane diminishes by unity the freedom of motion of all other such parts of the same metrical image.

3rd, when the particles and flat films of [a] metrical image are subject to such conditions as just suffice to reduce the freedom of motion to zero, there are just two positions, said to be *perverse* of one another, in which it can fulfill those conditions.

4th, if one metrical image or part of such image can occupy one place at one instant and another at another instant, then every such image or part of an image which can occupy the former place at one instant can occupy the latter place at another instant.

312. *Definition.* Two places (whether the same or different) are said to be equal which can be occupied by the same metrical image or by the same part of one metrical image. And the order of the occupied parts makes no difference.[†18]